To study the resonance condition of a series LCR circuit.

Manual as a Part of B. Tech Physics Laboratory

August 12, 2016
1 **Objective:**

To study the resonance condition of a series LCR circuit, and determine its quality factor (Q), bandwidth (BW) for different values of the resistor.

2 **Apparatus:**

1. Coil 600 turns, 9mH, Maximum current = 2 Amps
2. Resistors 47 Ω, 100 Ω and 1000 Ω; each of 1 W.
3. Capacitor 4.7 μF/160 V
4. Connecting cables, 4 mm plug, 32 A, red, 1 = 25 cm
5. Connecting cables, 4 mm plug, 32 A, blue, 1 = 25 cm
6. Connection box
7. Digital Multimeter
8. Digital function generator

3 **Procedure and Observations:**

![Circuit Diagram](image)

Fig.1 The circuit diagram for the LCR setup along with the vector diagram for the AC voltages/currents across different circuit elements. For detailed explanation on the vector diagram see section ‘Theory’.

1. **Note:** Please look at the section named ‘Precautions’ before starting the experiment. Otherwise there is a possibility of damaging the equipment/circuit elements.
2. Connect the circuit as shown in Fig. 1.
3. Switch on the power supply.
4. For the digital function generator, select the following settings:

   Function generator:
   
   i. DC-offset: ± 0 Volts (to be measured via multimeter in DC-mode)
ii. Amplitude: 3 to 4 V\text{RMS} (to be measured via multimeter in AC-mode). Instead of above one can also try to keep peak-to-peak voltage for the AC input fixed at 6 V within the function generator all throughout the measurements which will ensure the voltages across the electrical circuit elements to remain within ‘safety limits’.

iii. Frequency: 0-30 kHz

iv. Mode: sinusoidal

v. The display of the Digital function generator should be kept in kHz/Frequency mode.

5. Multimeter is set to AC mode.

6. Set appropriate range of the voltage (in Volts) on multimeter dial.

7. Keep the inductor and capacitor at constant values. Choose a particular value for the resistor. Measure the values of voltages V_R across the resistor for different values of the frequencies of the AC input. At the resonance condition (i.e. at f_{res}) there will be maximum voltage across resistor (see vector diagram in Fig.1 as well as section named ‘Theory’). Tabulate (in Table 1) the observations of frequencies and the corresponding voltages across resistor V_R (for given R).

Table 1: Observations (Freq. vs voltage)

Inductance = ….mH; Capacitance = ….\mu F

<table>
<thead>
<tr>
<th>Resistance (\Omega)</th>
<th>Frequency (f) (Hz)</th>
<th>Voltage (V_R) (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R=fixed value</td>
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8. Plot a graph of the voltage \(V_R\) vs. frequency \(f\). By locating the peak position of the graph, the resonance frequency of series LCR circuit \(f_{res}\) can be deduced. Here \(f_{res}\) is the frequency at the peak of the voltage curve (see section ‘Theory’; see also section ‘Precautions’).

9. At \(f_{res}\), measure \(V_L\), \(V_C\) along with \(V_R\) and tabulate the observations of the voltages and \(f_{res}\) as shown in Table 2.
10. Repeat measurements (point 7, 8 and 9) for each values of the resistor (47 Ω, 100 Ω and 1000 Ω). Compare the $V_R$ vs. frequency curves for different values of the resistor.

**Table 2: Observations**

<table>
<thead>
<tr>
<th>$V_R$ (V)</th>
<th>$V_L$ (V)</th>
<th>$V_C$ (V)</th>
<th>$f_{res}$ (Hz)</th>
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4 **Results:**

1. The quality factor ($Q$) is given by:

$$Q = \frac{X_L}{R} = \frac{2\pi f_{res}L}{R} \quad \text{..........................(1)}$$

2. Insert the value of $L$ and $f_{res}$ into above Eq. (1) and deduce the quality factor of the series resonant LCR circuit for different values of resistor.

3. The bandwidth can be calculated as $BW = f_{res}/Q$

4. Make a comparison table for the estimated values of bandwidth and quality factor for different resistors. Compare them and draw conclusions.

**Table 3: Comparison table**

<table>
<thead>
<tr>
<th>Resistance ($\Omega$)</th>
<th>Bandwidth (BW)</th>
<th>Quality factor ($Q$)</th>
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</table>
5 Precautions:

1. The connections should be tight.
2. Correctly set the digital function generator and multimeter.
3. Ensure the values of voltage and current are within the prescribed limits. Ensure that the wattages of resistors are not exceeded. Similarly ensure that the maximum permissible voltage rating for the capacitor is not exceeded.
4. Near $f_{\text{res}}$ take readings for smaller steps in frequency in order to find the exact value of the maximum voltage $V_{\text{Rmax}}$ and the frequency $f_{\text{res}}$ at which resonance occurs.
5. Select appropriate values of inductor, resistor and capacitor for the experiment.

6 Theory:

Definitions:

An LCR circuit is an electrical circuit consisting of a resistor (R), an inductor (L), and a capacitor (C), connected in series or in parallel.

The circuit forms a harmonic oscillator for current, and resonates in a similar way as an LC circuit. Introducing the resistor increases the decay of these oscillations, which is also known as damping. The resistor also reduces the peak resonant frequency. Some resistance is unavoidable in real circuits even if a resistor is not specifically included as a component.

Resonance:

An important property of this circuit is its ability to resonate at a specific frequency, the resonance frequency, $f_{\text{res}}$ (or $\omega_{\text{res}} = 2\pi f_{\text{res}}$). Resonance occurs because energy is stored in two different ways: in an electric field as the capacitor is charged and in a magnetic field as current flows through the inductor. Energy can be transferred from one to the other within the circuit and this can be oscillatory. A mechanical analogy is a weight suspended on a spring which will oscillate up and down when released. A weight on a spring is described by exactly the same second order differential equation as an LCR circuit and for all the properties of the one system there will be found an analogous property of the other. The mechanical property answering to the resistor in the circuit is friction in the spring/weight system. Friction will slowly bring any oscillation to a halt if there is no external force driving it. Likewise, the resistance in an LCR circuit will "damp" the oscillation, diminishing it with time if there is no driving AC power source in the circuit.
The resonance frequency is the frequency at which the impedance of the circuit is at a minimum. Equivalently, it can be defined as the frequency at which the impedance is purely resistive. This occurs because the impedances of the inductor (\(X_L\)) and capacitor (\(X_C\)) (also called as the reactances of inductor and capacitor) at resonance are equal but of opposite sign and cancel out. The formulae are \(X_L = \omega L\) and \(X_C = 1/\omega C\). Since, in an AC circuit, the resistances/reactances carry a definite phase relationships w.r.t. each other. They are conveniently represented by a vector notation in an effective 2-D plane. The direction of the vector gives the phase of the corresponding quantities. In this representation the vector for \(X_L\) is at an angle +90° w.r.t. the vector for \(R\) whereas the vector for \(X_C\) is at an angle -90° w.r.t. the same. Thus the angular difference between the vectors for \(X_L\) and \(X_C\) is 180° which tend to cancel them out. At resonance \(X_L = X_C\), where complete cancellation between \(X_L\) and \(X_C\) occurs. Hence \(\omega_{res}L = 1/\omega_{res}C\) giving rise to \(\omega_{res} = 1/\sqrt{LC}\).

**Damping:**

Damping is caused by the resistance in the circuit. It determines whether or not the circuit will resonate naturally (that is, without a driving source). Circuits which will resonate in this way are described as underdamped and those that will not are overdamped.

**Bandwidth:**

A key parameter concerning LCR circuit is bandwidth. The bandwidth is measured between the frequencies at which the power passed through the circuit has fallen to half the value passed at resonance. There are two of these half-power frequencies, one above, and one below the resonance frequency. Then we have \(\Delta \omega = \omega_2 - \omega_1\), where \(\Delta \omega\) is the bandwidth, \(\omega_1\) is lower half power frequency and \(\omega_2\) is upper half power frequency.

**Q-factor:**

The Q-factor is a widespread measure used to characterize resonators. It is defined as the peak energy stored in the circuit divided by the average energy dissipated in it per radian at resonance. Low Q circuits are therefore damped and lossy and high Q circuits are underdamped. Q is related to bandwidth; low Q circuits are wide band and high Q circuits are narrow band. \(Q = \omega_{res}/\Delta \omega\). For a LCR circuit it can be shown that Q is given by \(Q = (1/R)^*\sqrt{L/C}\).

As an example for a case of the circuit parameters (for the LCR circuit of Fig. 1):

- \(f = 10\) Hz
- \(E = 100\) V RMS
- \(L = 2\) H
- \(C = 0.0003\) F
- \(R = 10\) Ω
The measured voltages are as follows:

\[ V_L = 171.66 \text{ V} \]
\[ V_C = 72.6 \text{ V} \]
\[ V_R = 13.68 \text{ V} \]

The Vector representation of AC voltages/currents:

We must take into account the different phase angles between voltage and current for each of the three components in the circuit. The vector diagram to the right illustrates this concept (see Fig.1).

Since this is a series circuit, the same current passes through all the components and therefore our reference is at a phase angle of 0°. This is shown in red in the diagram. The voltage across the resistor, \( V_R \), is in phase with the current and is shown in green. The blue vector shows \( V_L \) at +90°, while the gold vector represents \( V_C \), at -90°. Since they oppose each other diametrically, the total reactive voltage is \( V_L - V_C \). It is this difference vector that is combined with \( V_R \) to find total voltage \( V_T \) (shown in cyan in the diagram).

We know that \( V_T = 10 \text{ V RMS} \). Now we can see that \( V_T \) is also the vector sum of \((V_L - V_C)\) and \( V_R \). In addition, because of the presence of \( R \), the phase angle between \( V_T \) and "i" will be \( \arctan((V_L-V_C)/V_R) \), and can vary from -90° to +90°.

The Mathematics:

Voltage and current calculation for this circuit are based on Ohm's Law. Our basic expressions are:

\[ i_L = \frac{V_L}{X_L} \]
\[ i_C = \frac{V_C}{X_C} \]
\[ i_R = \frac{V_R}{R} \]
\[ i = \frac{V}{Z} \]

Since this is a series circuit, the value of "i" in each expression is the same. That is

\[ i_L = i_C = i_R = i \]

and we can use \( i \) as our reference value for our calculations. We will also need to know the value of \( \omega (= 2\pi f) \) to determine \( X_L \) and \( X_C \). For \( f = 10 \text{ Hz} \),

\[ \omega = 2\pi f = 62.831853 \]

Now we can complete our calculations, starting with \( X_L \), \( X_C \), and \( Z \):
Allowing for calculator round-off errors through all these calculations, the total voltage \( V \) is exactly 100 volts, which is what we initially specified. Therefore our calculations check out and our results are valid.

**Resonance and the effect of \( R \):**

\[
X_L = \omega L \approx 125.6637 \ \Omega
\]

\[
X_C = \frac{1}{\omega C} \approx 53.0618 \ \Omega
\]

\[
Z = ((X_L - X_C)^2 + R^2)^{1/2} \approx 73.2874 \ \Omega
\]

\[
i = \frac{V}{Z} \approx 1.37 \ \text{A}
\]

\[
V_L = i \times X_L \approx 171.4 \ \text{V}
\]

\[
V_C = i \times X_C \approx 72.4 \ \text{V}
\]

\[
V_R = i \times R \approx 13.6 \ \text{V}
\]

\[
V = ((V_L - V_C)^2 + V_R^2)^{1/2} \approx 99.9288 \ \text{V} \approx 100 \ \text{V}
\]

At very low frequencies, capacitor \( C \) will be an open circuit, and virtually no current will flow through the circuit. At very high frequencies, inductor \( L \) will be an open circuit, and again no current will flow. However, at intermediate frequencies, both \( X_C \) and \( X_L \) will be moderate, and the difference between them will be small. At
resonance, that difference will be zero, and only R will limit the current flow in the circuit.

The above graph shows normalized values of current through a series RLC circuit at frequencies ranging from 0.01 times the resonant frequency, to 100 times that frequency. Beyond that range, as you can see from the graph, no significant current will flow at all. Within that range, current depends primarily on the value of R.

In a fully normalized circuit, R = 1Ω. This will permit a current of 1 A to flow at resonance, as indicated by the green curve on the graph. Similarly, if we set R as 2Ω, the current will be 0.5 A at resonance. The blue curve shows this.

The remaining curves show what happens if we reduce R. The yellow curve shows current when R = 0.5Ω, while the red curve is for R = 0.1Ω.

Note that for low values of R, the current at resonance peaks much higher, but falls off very quickly as frequency changes. For higher values of R, the curve is much broader, and maximum current remains much lower. This is the standard trade-off between bandwidth and maximum current, and the value of R is critical for controlling this factor.

**Changing the Ratio of L/C:**

When we change the ratio of L/C, we change the reactance of both components at any given frequency without changing the resonant frequency. This is accomplished by making sure that the product of L and C remains constant even when we change their ratio. Thus, if L = 1H and C = 1F, LC = 1 and L/C = 1. However, if L = 2H and C = 0.5F, we still have LC = 1, but now L/C = 4. Or, if L = 0.5H and C = 2F, L/C = 0.25.

By changing L and C in this manner, we change the values of XL and XC at and near the resonant frequency without changing the resonant frequency itself. This controls the overall impedance of the circuit at frequencies near resonance and gives the resistor, R, either more or less control over current at those frequencies. The result is a change in the frequency range over which this circuit will conduct significant amounts of current. The following three graphs illustrate this:
When we plot the curves this way, it becomes clear that as we increase the L/C ratio, we limit the circuit to passing current over an increasingly narrow band of frequencies. However, as we reduce the L/C ratio, we widen the frequency band over which this circuit will pass significant amounts of current. This becomes very important when we deal with certain types of filters, and especially with tuned circuits.

References:

2. [http://vlab.amrita.edu/?sub=1&brch=75&sim=330&cnt=1](http://vlab.amrita.edu/?sub=1&brch=75&sim=330&cnt=1)